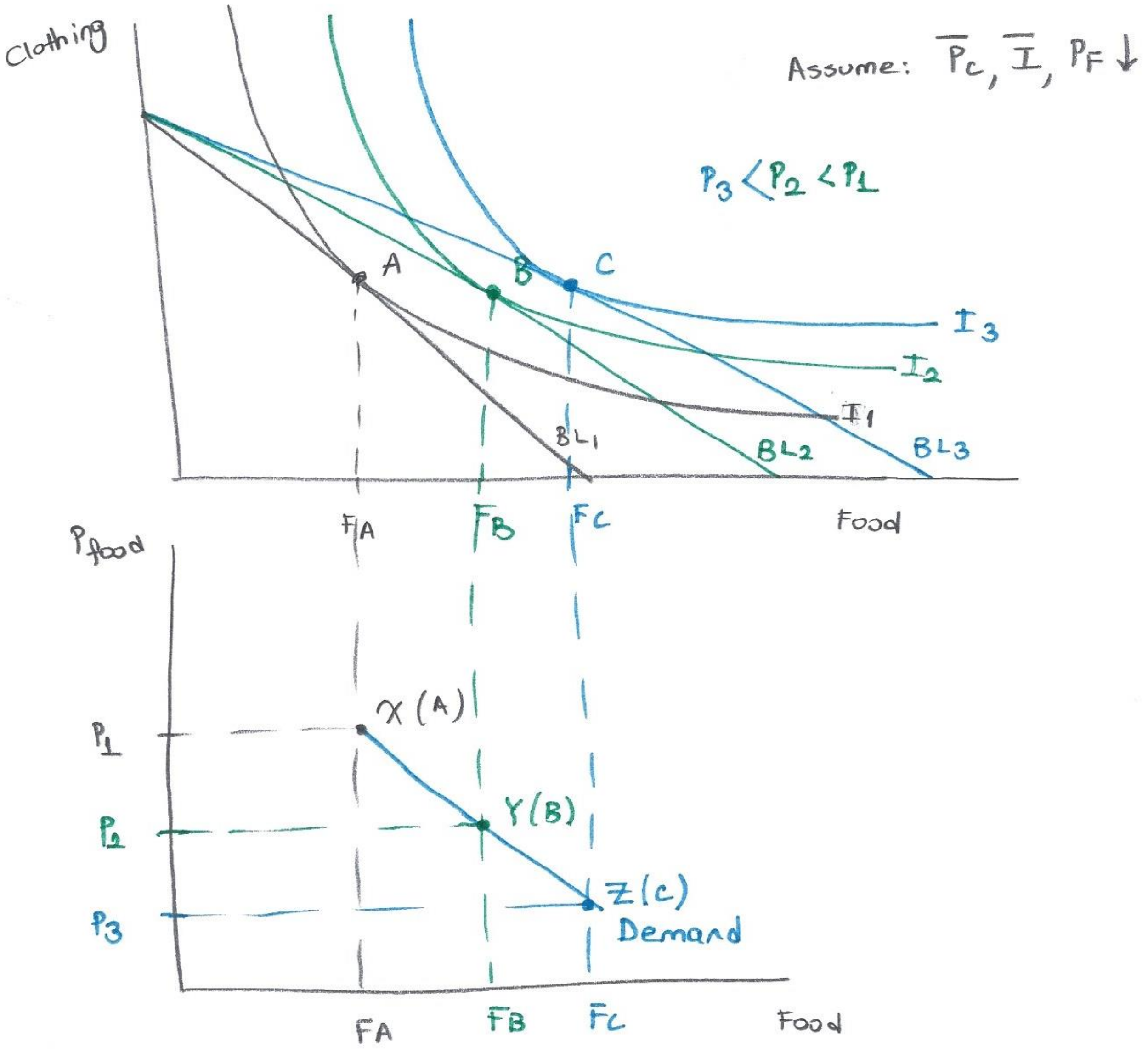
Lecture Notes:

Derivation of the demand curve from price-consumption line



Deriving the demand curve :

$u(X,Y) = XY \quad , \quad MU_X = \frac{\partial U}{\partial X} = Y \quad , \quad MU_Y = \frac{\partial U}{\partial Y} = X$

Budget constraint $P_X \cdot X + P_Y \cdot Y = I$

$$\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$$

$$\frac{Y}{X} = \frac{P_X}{P_Y}$$

$$\boxed{Y = \frac{X \cdot P_X}{P_Y}}$$

$$P_X \cdot X + P_Y \cdot Y = I$$

$$P_X \cdot X + \cancel{P_Y}\left(\frac{X \cdot P_X}{\cancel{P_Y}}\right) = I$$

$$2 P_X \cdot X = I$$

$$\boxed{X^* = \frac{I}{2P_X}}$$

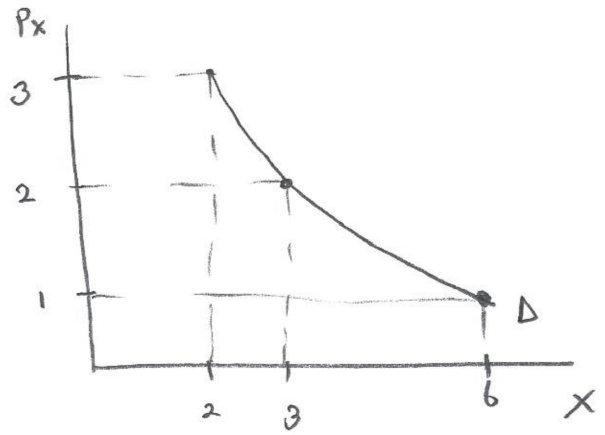
$P_X \uparrow$ from \$1 to \$2 to \$3 , $P_Y = 1$, $I = 12$

$$\boxed{X^* = \frac{12}{2 \cdot P_X} = \frac{6}{P_X}}$$

$X(1) = 6 \rightarrow 6/1$

$X(2) = 3 \rightarrow 6/2$

$X(3) = 2 \rightarrow 6/3$

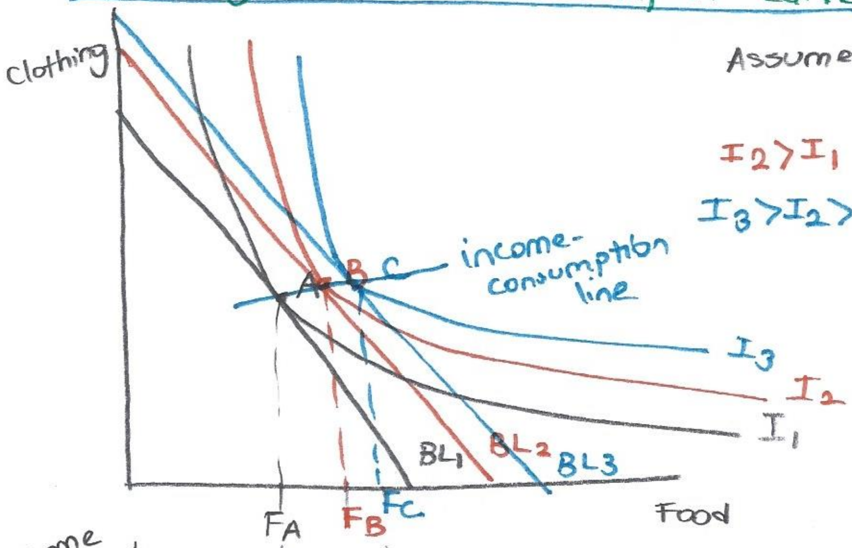


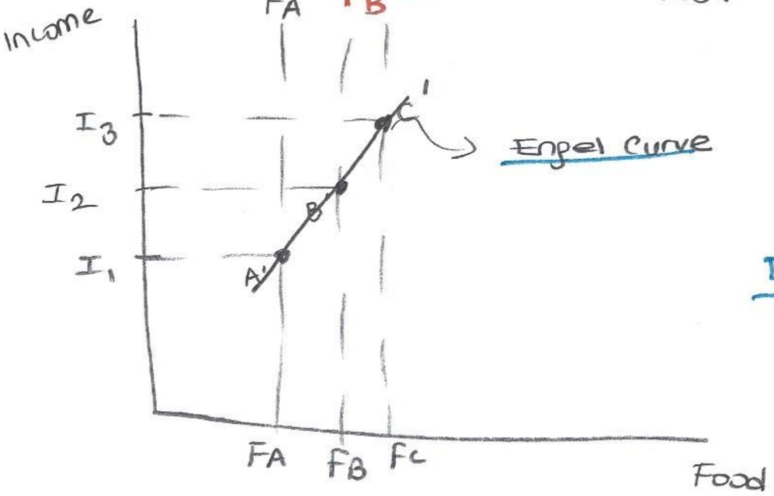
Deriving the income-consumption curve

Assume: $\bar{P}_F$, $\bar{P}_C$, $I\uparrow$

$I_2 > I_1$

$I_3 > I_2 > I_1$

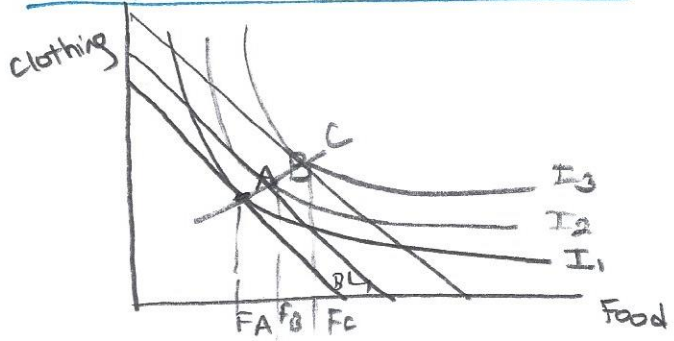


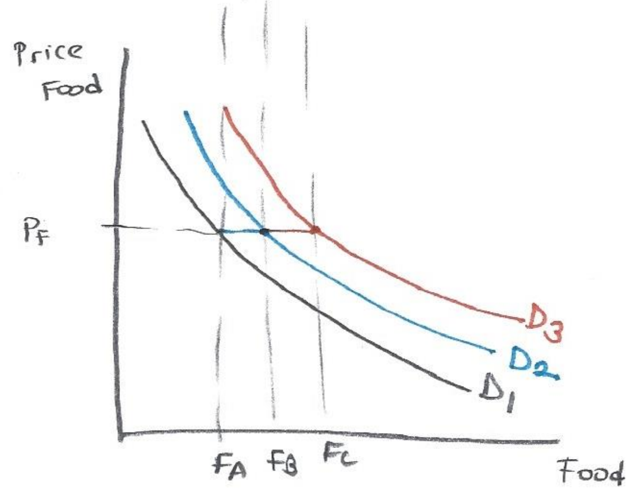


income-consumption curve slopes (+) then Engel curve slopes (+)

Derivation of the Engel curve

shifts of the Demand Curve



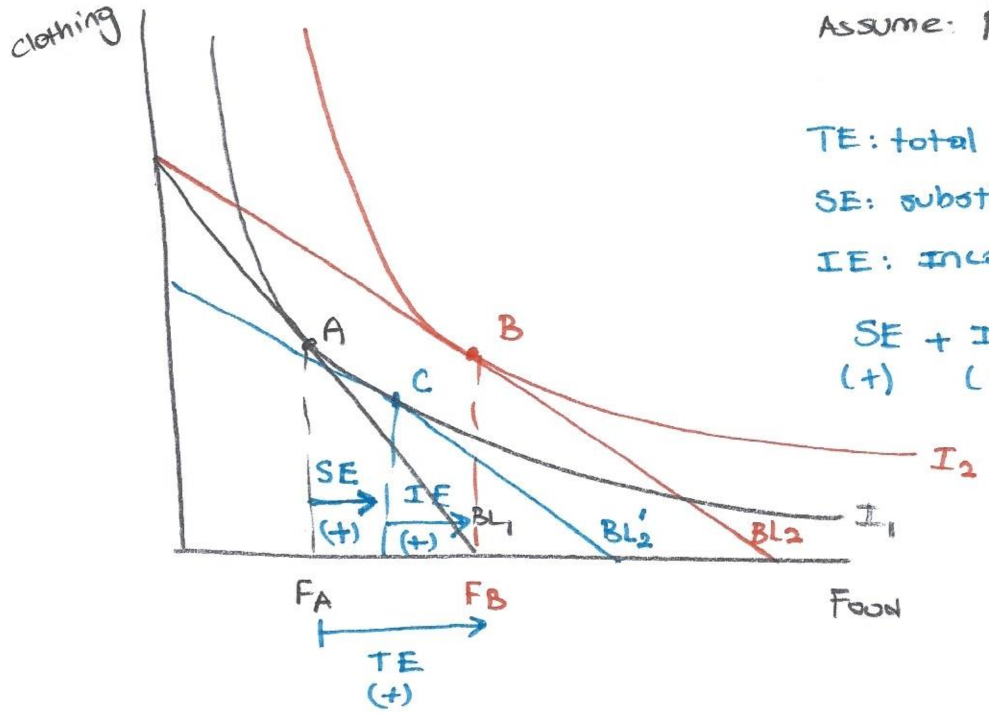


For Normal Good

$Y\uparrow \Rightarrow D\uparrow$ (shifts to the right)

Income and Substitution Effects

FOR NORMAL GOODS



Assume: $\bar{P_C}$, $\bar{I}$, $P_F\downarrow$

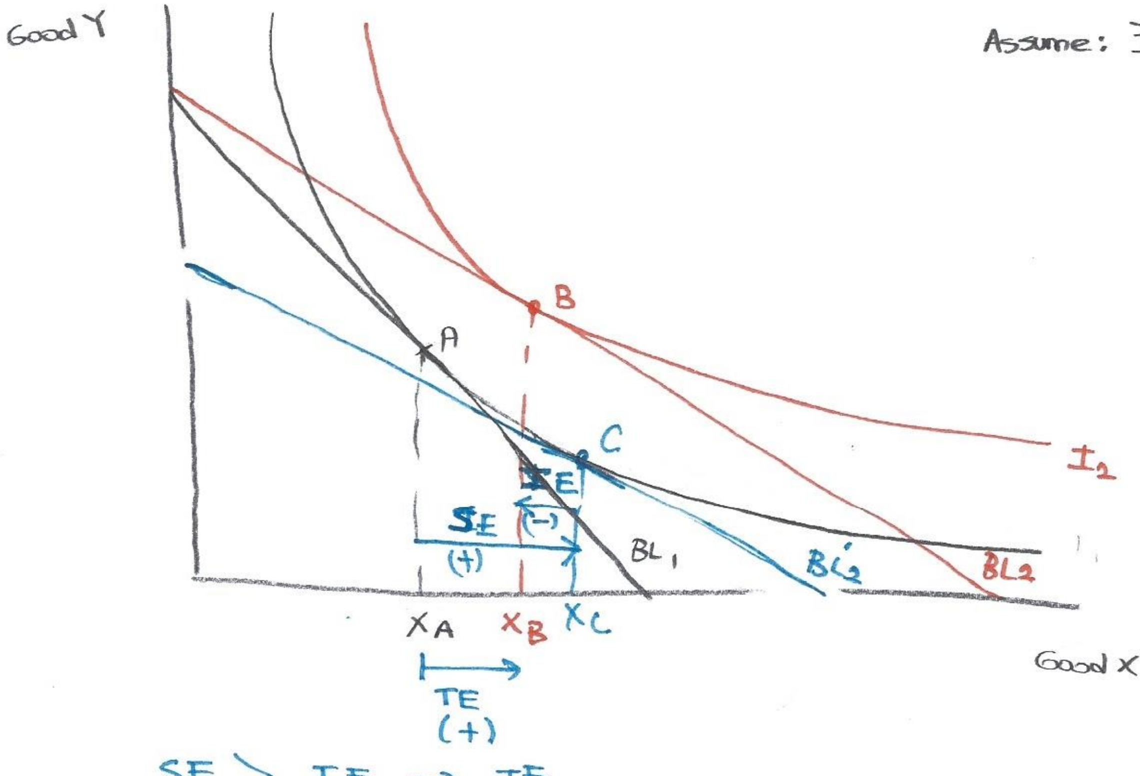
TE: total effect

SE: substitution effect

IE: income effect

$\underset{(+)}{SE} + \underset{(+)}{IE} \Rightarrow \underset{(+)}{TE}$

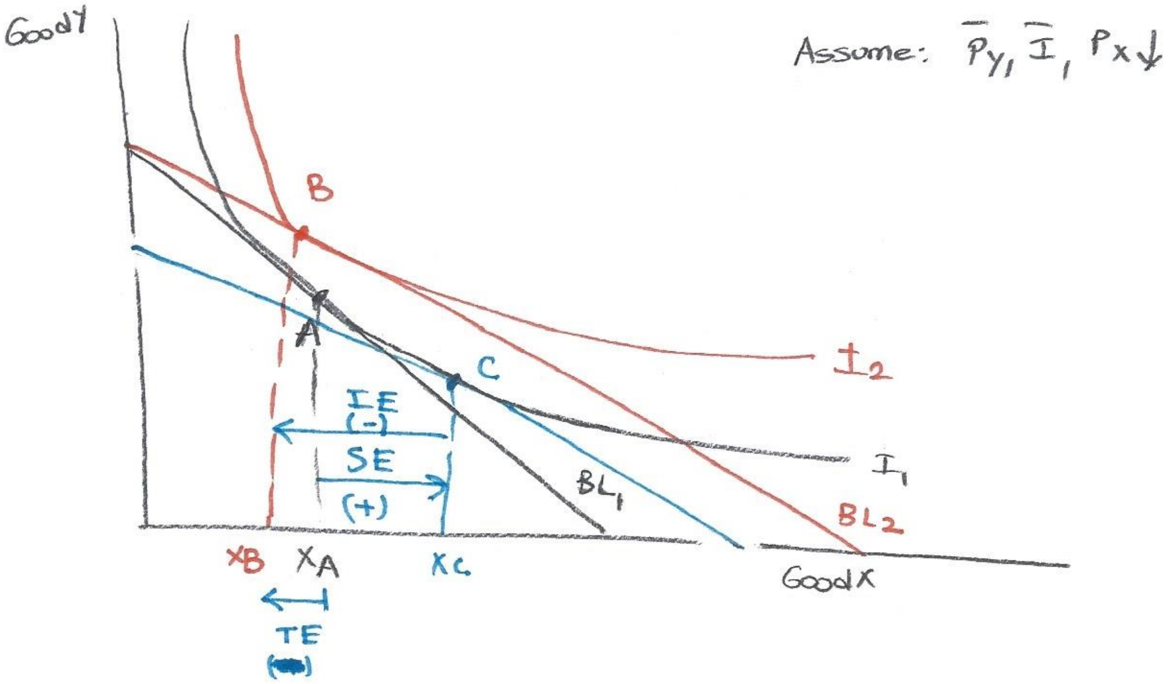
FOR INFERIOR GOODS



Assume: $\bar{I}$, $\bar{P_Y}$, $P_X\downarrow$

$\underset{(+)}{SE} > \underset{(-)}{IE} \Rightarrow \underset{(+)}{TE}$

FOR GIFFEN GOODS

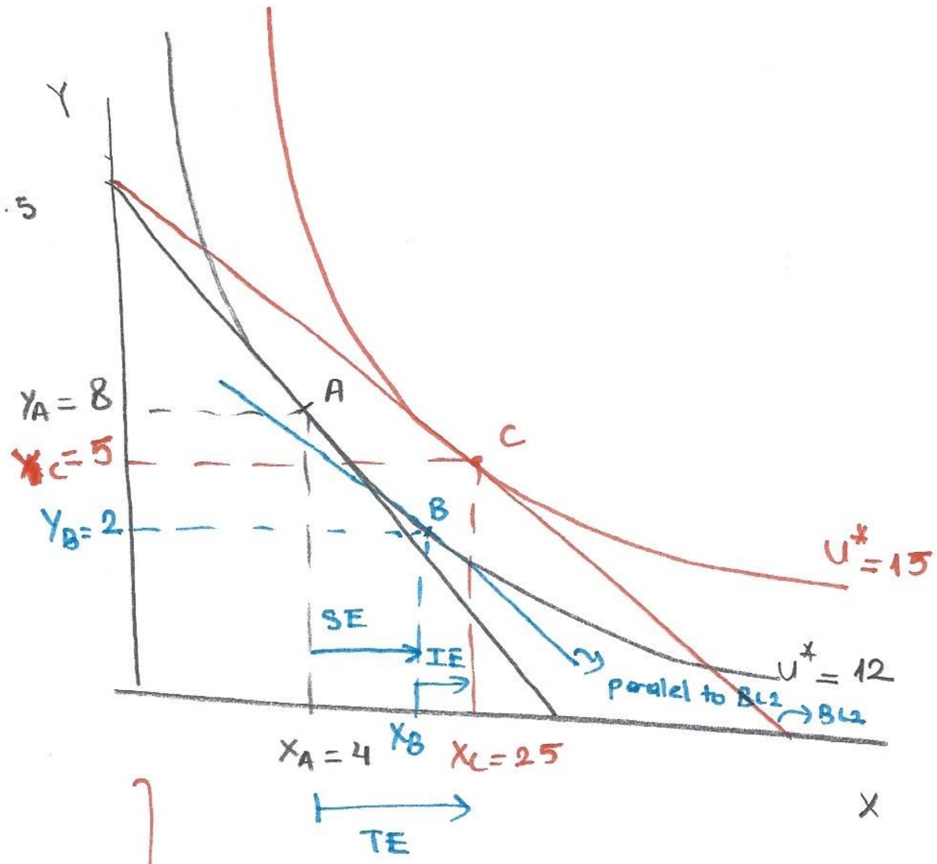


$$\underset{(-)}{IE} > \underset{(+)}{SE} \Rightarrow \underset{(-)}{TE}$$

Finding Income and Substitution Effects

$u(x,y) = 2x^{0.5} + Y$

$P_y = 1, \quad I = 10$



① Suppose $P_x = 0.5$

$MU_x = \frac{\partial U}{\partial x} = 2(0.5)x^{-0.5} = x^{-0.5}$

$MU_y = \frac{\partial U}{\partial y} = 1$

$\frac{MU_x}{MU_y} = \frac{P_x}{P_y}$

$\frac{x^{-0.5}}{1} = \frac{0.5}{1}$

$\frac{1}{x^{0.5}} = 0.5$

$(x^{0.5})^2 = \left(\frac{1}{0.5}\right)^2 = \left(\frac{1}{\frac{1}{2}}\right)^2 = 4$

$\boxed{X_A = 4}$

$P_x \cdot X + P_y \cdot Y = I$

$(0.5)4 + 1 \cdot Y = 10$

$\boxed{Y_A = 8}$

$u^* = 2 \cdot x^{0.5} + Y = 2 \cdot 4^{0.5} + 8$

$\boxed{U^* = 12}$

② $P_x \downarrow \Rightarrow P_{x\,new} = 0,2$

$\frac{x^{-0.5}}{1} = \frac{0.2}{1}$

$x^{-0.5} = 0.2$

$\frac{1}{x^{0.5}} = 0.2$

$(x^{0.5})^2 = \left(\frac{1}{0.2}\right)^2 = (5)^2$

$\boxed{X_C = 25}$

$P_x X + P_y \cdot Y = I$

$(0.2)25 + 1 \cdot Y = 10$

$5 + Y = 10$

$\boxed{Y_C = 5}$

$u^* = 2 \cdot 25^{0.5} + 5$

$\boxed{u^* = 15}$

$\Rightarrow$

Try to find X_B

$\frac{MU_x}{MU_y} = \frac{P_x}{P_y}$

$\frac{x^{-0.5}}{1} = \frac{0.2}{1} \Rightarrow X_B = 25$

u^* must be $u^* = 12$ (orijinal)

$U = 2X^{0.5} + Y$

$2 \cdot 25^{0.5} + Y = 12$

$2 \cdot 5 + Y = 12$

$\boxed{Y_B = 2}$

Substitution effect $= SE = X_B - X_A = 25 - 4 = 21$

Income effect $= X_C - X_B = 0$

Total effect $= X_C - X_A = 25 - 4 = 21$ //

Example: $U(X,Y) = X^2 + Y$

$P_y = 10$, $I = 500$

① suppose $P_x = 40 \Rightarrow$

$$\frac{MU_X}{MU_Y} = \frac{P_X}{P_y}$$

$$\frac{2X}{1} = \frac{\cancel{40}^{\,2}}{\cancel{10}}$$

$$\boxed{X_A = 2}$$

$$P_x \cdot X + P_y \cdot y = I$$

$$40 \cdot 2 + 10 \cdot Y = 500$$

$$80 + 10Y = 500$$

$$10Y = 420$$

$$\boxed{Y_A = 42}$$

$$U^* = X^2 + Y = 4 + 42 = 46 //$$

② $P_X \downarrow \Rightarrow P_x = 20 \Rightarrow$

$$\frac{2X}{1} = \frac{20}{10}$$

$$\boxed{X_C = 1}$$

$$P_x \cdot x + P_y \cdot y = I$$

$$20 \cdot 1 + 10 \cdot Y = 500$$

$$20 + 10Y = 500$$

$$10Y = 480$$

$$\boxed{Y_C = 48}$$

$$\boxed{U^* = X^2 + Y = 1 + 48 = 49}$$

(Maybe Giffen)

We must take this budget line which is parallel to BL_2

$$\boxed{X_B = 1}$$

U^* must be the original one

$$\boxed{U^* = 46}$$

$$46 = X^2 + Y$$

$$46 = 1 + Y$$

$$\boxed{Y_B = 45}$$

Substitution effect $= X_B - X_A$

$= 1 - 2 = -1$

(−) SE

Income effect $= X_C - X_B$

$= 1 - 1 = 0$

Total effect $= X_C - X_A$

$= 1 - 2 = -1$

TE(−)